

An Efficient and Accurate Rough Set for Feature Selection, Classification and Knowledge Representation

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Abstract—This paper presents a strong data mining method based on the rough set, which can realize feature selection, classification and knowledge representation at the same time. Although the rough set, a popular method for feature selections, has a good interpretability, its low efficiency and low accuracy prevent it from immediately being applied to real-world scenarios. To address the efficiency issue of the rough set, we discover the stability of the local redundancy (SLR) of attributes and propose the theorem to rigorously prove it. In regards to the accuracy issue, we first show that overfitting leads to the ineffectiveness of rough sets, especially when processing noise attributes. We then propose relative importance, a robust measurement for an attribute to alleviate such overfitting issues. In this paper, we propose a novel rough set framework, which significantly improves the efficiency and accuracy upon existing rough set methods. We further develop our rough set framework by proposing a "rough concept tree" concept for knowledge representation and classification. Experimental results on public benchmark data sets show that our proposed framework achieves higher accuracies than seven popular rough set methods and other state-of-the-art feature selection methods.

Index Terms—rough set, attribute reduction, feature selection, acceleration framework

I. INTRODUCTION

FEATURE selection, also called as attributed selection, is a foundational process in data preprocessing. Not only reducing the difficulty of learning, it also alleviates the disaster of dimensionality. Due to these advantages, past decades have witnessed rapid developments in this promising direction. To be more specific, there are three major research directions regarding feature selection: global optimal search, random search and heuristic search according to the formation process of the search strategy subset. The branch and bound method, which searches the optimal subset when the feature number in the optimal feature subset is determined in advance, is the only global search method capable of obtaining the optimum [1]. In contrast, the random search strategy assigns a certain weight to each feature during the operation of the algorithm. For example, the relief series algorithm is a typical random

search method that selects features according to the weight. This simple-yet-effective algorithm, although possesses broad applications, is unable to remove the redundancy and only be applicable to two classification types [2]. On the other hand, the floating generalized backward selection method in heuristic search is a feature selection search strategy that is more conducive to practical applications. It not only considers the statistical characteristics between features, but also ensures the speed and stability of the algorithm operation [3]. The sound efficiency of the heuristic search strategy comes at a cost of sacrificing the accuracy regarding the global optimum. According to whether a selection method is independent of the learning algorithm, it can be divided into three categories — filter method, wrapper method and embedded method. The main idea of the filter method is to score the features, and then sort the features in a descending order according to their scores. The wrapper method recognizes feature selection as a feature subset optimization process, and uses classifiers to evaluate the feature subset. Since each feature subset needs to train a classifier, most wrapper methods are inefficient. Besides, limited by the classifier used by this method, the feature subset obtained by wrapper methods tends to have lower versatility. Therefore, prior works of wrapper method mainly focuses on the optimization process, motivating the embedded method, where the feature selection is embedded into the training process.

Feature selection based on the rough set theory belongs to the filter method. Rough set theory, an effective mathematical tool to process uncertain, inconsistent and incomplete data, was first proposed by a Polish scientist, Pawlak, in 1982 [2]. It has been widely applied in machine learning, data mining and decision support systems [3]–[9]. Since the forward heuristic attribute selection algorithm based on the rough set theory can effectively reduce the time complexity, past decades witnessed rapid developments in its related fields [10]–[16]. To elaborate, Miao and Hu redefined the evaluation criteria of attributes in a decision system from the perspective of mutual information using the attribute importance as a heuristic search strategy to reduce the search space during the attribute reduction [17]. Wang, Yu, and Yang designed a heuristic knowledge reduction algorithm for decision tables using conditional information entropy as the heuristic. They performed a careful analysis for informative and algebraic definitions in the rough set theory [18]. Liang, Chin, Dang, and Yam introduced a new definition of information entropy and the concept of complementary information entropy [19]. They validated that their metric can

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be categorized as a fuzzy entropy, which can be used to measure the fuzzy degree and rough classification of rough sets. They also proposed a heuristic algorithm based on complementary information entropy. It is worth noting that all of these methods, which are designed for specific scenarios, cannot be used to accelerate rough set as a general method. In contrast, Qian and Liang [32] proposed the only general method to accelerate the rough set by introducing the positive region acceleration and pre-calculation for core attributes.

In fact, the efficiency of rough set is yet to be efficient and accurate enough. Besides, as far as we know, there are few studies being capable of improving the accuracy of rough sets for a wide range of applications. These shortcomings state the major obstacles of immediate real-world deployments for an algorithm based on the rough sets. In order to address such issues, we present the following contributions in this paper:

(1) We propose a rigorously proved theorem to elaborate the stability of the attribute local redundancy in a rough set, which can be used to considerably accelerate most of the existing rough set algorithms.

(2) We discover that overfitting, especially when in a type of noise attribute, accounts for the ineffectiveness of rough sets. Our brand-new measurement increases the robustness of the rough set to the type of noise attribute with an improved generalizability while significantly enhancing its accuracy.

(3) We propose a novel general framework, RSLRS, which is not only able to considerably accelerate most of the rough set methods, but enhance their accuracies as well.

(4) We present the concept of "rough concept tree" for knowledge representations and classifications, enabling algorithms based on the rough set to effectively implement feature selection, data classification and knowledge representation simultaneously, rather than relying on any other classifiers.

The main contents of subsequent chapters are organized as follows: Chapter II lists the related work regarding the basic models of rough sets; Chapter III details the newly proposed accelerated rough set model based on the attribute local redundancy; Chapter IV discusses the model of "relative importance" which can improve accuracy of feature selection of rough sets; in chapter V, an efficient and accurate rough set framework is proposed based on the proposed theorems and model; in Chapter VI, we develop the rough set so that it can be effectively used for knowledge representation and data classification; the experimental results and analysis are presented in Chapter VII; We conclude our paper in Chapter VIII.

II. RELATED WORK

A. Filter Methods

The essence of filter methods is to use some indicators in mathematical statistics to score features, such as Pearson correlation coefficient, Gini coefficient, Kullback–Leibler divergence, Fisher score, Similarity measure, and etc. Since the filter method only uses the dataset rather than relying on a specific classifier, it has strong versatility and is easy to expand. Compared with wrapper methods and embedded methods, filter methods have lower algorithm complexity. However, at

the same time, the classification accuracy of filter methods is usually lower than the other two methods. In addition, because filter methods only score a single feature rather than the whole feature subset, the feature subset obtained by filter methods usually has a high redundancy.

G. Qu et al [20] proposed a generalized Fisher score feature selection method, which aims at finding a feature subset to maximize the lower bound of Fisher score. This method transforms feature selection into a quadratically constrained linear programming and uses the cutting plane algorithm to solve this problem. G. Roffo et al proposed a feature selection method based on graph, which ranks the most important features by recognizing them as arbitrary clue sets [39]. This method maps a feature selection problem to an affinity graph by taking features as nodes, and then evaluates the importance of nodes by the eigenvector centrality. G. Roffo et al. also proposed a filtering feature selection method, Inf-FS [40], which takes features as nodes of a graph and views feature subset as a path in the graph. The power series property of matrix is used to evaluate the path, and the computational complexity is reduced by adding the paths until the length reaches infinity.

B. Wrapper Methods

Wrapper methods evaluate features using the performance of learning algorithms, so the classification accuracy of wrapper methods is often higher than that of filter methods. However, limited by the classifier used by this method, the feature subset obtained by wrapper methods tends to have a lower versatility. For each feature subset, wrapper methods need to train a classifier. Therefore, this method usually has a high computational complexity of, which depends on the search strategy of the feature subset. However, wrapper methods evaluate the entire feature subset, rather than a single feature, and consider the dependency between features, so the redundancy of the result feature subset is often lower than that of filter methods.

Support vector machine (SVM) is a popular learning algorithm of the wrapper method. I. Guyon et al proposed a feature selection method of support vector machine based on recursive feature elimination [23]. The idea of this method is to construct the ranking coefficient of features according to the weight vector generated by SVM during training. In each iteration, the feature with the smallest ranking coefficient is removed. At the end of this algorithm, a descending sorting of all the features is obtained. J. Guo et al proposed a feature selection method based on distance-based clustering, which uses a triplet-based ordinal locality preserving loss function to capture the local structures of the original data. Meanwhile, this method defines an alternating optimization algorithm based on half-quadratic minimization to speed up the optimization process of this algorithm [24]. In order to overcome the issue that only single feature is considered in the filtering method, a joint learning framework for feature selection and clustering, namely Dependence Guided Unsupervised Feature Selection (DGUFS), was proposed [25]. DGUFS is a projection-free feature selection model based on $L_{2,0}$ -norm equality constraints and defines two

dependence guided terms to enhance the dependence among original data, cluster labels and selected features.

C. Embedded Methods

Embedded methods embed feature selection as a component into the learning algorithm. The feature subset can be obtained when the training process of the learning algorithm is completed. The embedded method is similar to the filter method. However, it determines the score of features via model training, which is different from the filter method. The main idea of this method is to select features important to the model training when to determine the model. Meanwhile, an embedded method is compromise to a filter method and a wrapper method. Compared with the filter method, the embedded method can achieve a higher classification accuracy. When comparing to wrapper methods, embedded methods have a lower algorithm complexity and are more difficult to suffer from the overfitting issue.

P. Bradley et al proposed an embedded feature selection method based on concave minimization and support vector machine [26], which finds a separation plane to distinguish two point sets in the n-dimensional feature space which uses as few features as possible. This method minimizes the weighted sum of the distance between the wrong classification points and the boundary plane while maximizing the distance between the two boundary planes of the separation plane. Embedded methods are often based on regression learning algorithms. F. Nie et al [27] proposed an efficient and robust feature selection method, which uses a loss function based on $L_{2,1}$ -norms to remove the outliers. This method adopts joint $L_{2,1}$ -norms minimization on loss function and regularization and proposes an effective algorithm to solve a series of joint $L_{2,1}$ -norms minimization problems. Y. Yang et al proposed a feature selection method named Unsupervised Discriminative Feature Selection (UDFS) [28]. UDFS optimizes a $L_{2,1}$ -norm regularized minimization loss function, which utilizes discriminative information and local structure of data distribution.

D. Rough Set Theory

The rough set theory mainly focuses on the information granulation and approximation. An equivalence relation granulates the sample space into disjoint equivalence classes, or sample information granules. It uses upper and lower approximation bounds to characterize the uncertainty of the information granules in order to approximate the arbitrary knowledge in the sample space. Classic rough set theory is defined on the basis of a strict equivalence relation and an indiscernibility relation. In this section, in order to lay a foundation for our theorem and proof, we review some of the basic concepts of they rough set theory, which has been presented in our previous work [29]. Here we first introduce information systems and indistinguishable relations.

Definition 1. [29] An *information system* is a quarternion $\langle U, A, V, f \rangle$, where: $U = \{x_1, x_2, \dots, x_n\}$ denotes a non-empty finite set of objects. U is called the universe; $A = \{a_1, a_2, \dots, a_m\}$ denotes a non-empty finite set of attributes; $V = \bigcup_{a \in A} V_a$ denotes the set of all attribute values, where V_a

denotes the value range of attribute a ; $f = U \times A \rightarrow V$ denotes a mapping function. $\forall x_i \in U, a \in A$, we have $f(x_i, a) \in V_a$. If the set of attributes A in the information system above satisfies: $A = C \cup D, C \cap D = \emptyset, D \neq \emptyset$, where C is the conditional attribute set and D is the decision attribute set, then the information system is called the *decision system* $\langle U, C, D \rangle$.

Definition 2. [30] Let $\langle U, A, V, f \rangle$ be an information system. $\forall x, y \in U$ and $B \subseteq A$, the *indistinguishable relationship* $IND(B)$ of the attribute subset B is defined as

$$IND(B) = \{(x, y) \in U \times U \mid f(x, a) = f(y, a), \forall a \in B\}. \quad (1)$$

In fact, $(x, y) \in IND(B)$ shows that the values of samples x and y are completely consistent under the attribute subset B ; that is, under the description of the attribute subset B , the samples x and y are indistinguishable.

$IND(B)$ is symmetric, reflexive, and transitive; that is, $\forall B \subseteq A$, $IND(B)$ is an equivalence relation on U (abbreviated R_B). $IND(B)$ creates a partition of U , denoted $U/IND(B)$ and abbreviated U/B . The characteristics of U/B are as follows: Suppose $U/B = \{X_1, X_2, \dots, X_k\}$, if $X_i, X_j \subseteq U$, $X_i \cap X_j = \emptyset (i \neq j)$, $\bigcup_{i=1}^k X_i = U$, then U is divided into k parts by $IND(B)$ and they form a partition of U . An element $[x]_B = \{y \in U \mid (x, y) \in IND(B)\}$ in U/B is called an equivalence class. This leads us to our next set of definitions, approximations based on the equivalence relation R_B .

Definition 3. [30] Let $\langle U, A, V, f \rangle$ be an information system. $\forall B \subseteq A$ there is a corresponding equivalent relationship R_B on U . $\forall X \subseteq U$, the *upper and lower approximations of X* with respect to B are defined as follows:

$$\overline{R_B}X = \cup \{[x]_B \in U/B \mid [x]_B \cap X \neq \emptyset\}, \quad (2)$$

$$\underline{R_B}X = \cup \{[x]_B \in U/B \mid [x]_B \subseteq X\}. \quad (3)$$

The lower approximation $\underline{R_B}X$ represents the set of samples in U that are determined to belong to X according to the equivalence relation R_B . It essentially reflects the ability of the equivalence relation R_B to approximately describe the knowledge contained in X by a partition of the knowledge of the universe U . It is also commonly called as the *B positive region of X in U* , abbreviated as $POS_B(X)$ in the following contents of the paper.

Definition 4. [30] Let $\langle U, C, D \rangle$ be a decision system. We notate the partition of the universe U by the decision attribute set D into L equivalence classes by $U/D = \{X_1, X_2, \dots, X_L\}$. $\forall B \subseteq C$, there is a corresponding equivalent relationship R_B on U . The *upper approximation and the lower approximation of D with respect to B* are respectively defined as

$$\overline{R_B}D = \bigcup_{i=1}^L \overline{R_B}X_i, \quad (4)$$

$$\underline{R_B}D = \bigcup_{i=1}^L \underline{R_B}X_i. \quad (5)$$

Definition 5. [30] Let $\langle U, C, D \rangle$ be a decision system. $\forall B \subseteq C$, the *positive region* and *boundary region* of D with respect to B are respectively defined as:

$$POS_B(D) = \underline{R}_B D, \quad (6)$$

$$BN_B(D) = \overline{R}_B D - \underline{R}_B D. \quad (7)$$

The size of the positive region reflects the separability of the classification problem in a given attribute space. The larger the positive region, the more detailed the classification problem can be described with using this attribute set. We find it useful to describe it mathematically: the *dependence* of D on B is defined as

$$\gamma_B(D) = \frac{|POS_B(D)|}{|U|}, \quad (8)$$

where $|\cdot|$ is the cardinality of the set and $0 \leq \gamma_B(D) \leq 1$. Obviously, the larger the positive region, the stronger the dependence of D on B .

The dependency function defines the contribution of conditional attributes to a classification, so it can be utilized as an evaluation index for the importance of the attribute set.

Definition 6. [31] Given a decision system $\langle U, C, D \rangle$, for all $B \subseteq C$ and $a \in C - B$, the *importance* of a relative to B is defined as

$$SIG(a, B, D) = \gamma_{B \cup a}(D) - \gamma_B(D). \quad (9)$$

The most widely used rough set is forward heuristic. It is not only efficient but can generate different knowledge rules as well. Therefore, the forward heuristic rough set is set as the default method in this paper. To be more specific, it uses the measurement SIG in (9) to select attributes in a forward way. The selection result C' is initialized with \emptyset . For each attribute a in the attribute $C - C'$, the one with the largest value of $SIG(a, C', D)$ and is greater than 0 will be selected into C' . This process is repeated until there is no more attribute greater than 0 in $SIG(a, C', D)$.

III. SPEED UP ROUGH SET USING THE STABILITY OF LOCAL REDUNDANCY OF AN ATTRIBUTE

A. Motivation

In [32] and [34], three most important theorems for generally speeding up the rough set that can be described as follows:

Theorem 1 Given a decision system $\langle U, C, D \rangle$, for $B \subseteq C$, we have

$$POS_B(D) \subseteq POS_C(D). \quad (10)$$

That is, the positive region of a attribute set must belong to the positive region of its parent attribute set. According to Theorem 1, we deduce that for any given parent attribute sets, the positive region of a child attribute set is not needed in order to calculate the positive region of the parent attribute set. Therefore, a general rough set algorithm can be accelerated.

Theorem 2 Let $\langle U, C, D \rangle$ be a decision system. Let $R' \subseteq R \subseteq C$ and $a \in R$ be a given attribute, and let $R - R'$ be a relative reduction of R . If $POS_{R-a}(D) \neq POS_R(D)$, i.e., that a is a single non-redundant attribute, a is not a redundant

attribute in $R - R'$. In other words, a single non-redundant attribute is a core attribute.

Theorem 2 indicates that is that the non-redundant attributes relative to given attribute set is also the non-redundant attribute relative to its child attribute set. Therefore, the non-redundant attributes in the all attributes are not needed to be considered at any time. The calculation cost of attribute combination can be reduced accordingly. We find that, from the perspective of the stability, Theorem 1 locates the stability of positive region while Theorem 2 finds the stability of non-redundancy attribute. That is, a redundant attribute relative to the child attribute set under the conditions in Theorem 1 is also the redundant attribute relative to its parent attribute set. In other words, the redundancy of the attribute is stable. Therefore, this type of redundant attributes will not be considered in the later feature selection process of a rough set, such that the rough set can be accelerated.

Theorem 3 [34] **{The stability of redundancy (SR) attribute}**. Given a decision system $\langle U, C, D \rangle$, let $R \subseteq C$ and $a \in C$ is a given attribute. If $U/R = U/(R + a)$, a is a redundant attribute relative to R .

Theorem 3 describes a finding on redundancy attributes for further speeding up the rough set. However, the condition to decide whether a attribute is redundant or not is rigorous. Therefore, in some cases, one may not be able to observe a significant acceleration. In the following theorem, we find that, even for most of those non-redundant attributes, there exists the local redundancy so some of the calculations for the equivalence class can be avoided. The theorem related to the local redundancy is detailed in the next section.

B. The Stability of Local Redundancy of an Attribute for accelerating Rough Set

Inspired by Theorem 3, we describe the definitions of the active region and the non-active region.

Definition 7. {Active Region and Non-Active Region}. Let $\langle U, C, D \rangle$ be a decision system. Let $R \subseteq C$ and $a \in C$ (but $a \notin R$) be a given attribute. The equivalence class that U is divided into under the attribute set R is $U/R = \{X_1, X_2, \dots, X_i, X_{i+1}, \dots, X_l\}$ and the equivalence class that U is divided into under the attribute a is $U/a = \{X'_1, X'_2, \dots, X'_k, X'_{k+1}, \dots, X'_s\}$. For $\forall j \in 1, 2, \dots, i$, if $X_j \subseteq X'_t$ ($t = 1, 2, \dots, s$) exists, then we define set $NACT_a(R) = X_1 \cup X_2 \cup \dots \cup X_i$ as the non-active region of attribute a relative to R and set $ACT_a(R) = X_{i+1} \cup \dots \cup X_l$ as the active region of the attribute a relative to R .

Based on Definition 7 and Theorem 3, a better acceleration method is presented in Theorem 4.

Theorem 4 {The stability of local redundancy (SLR) of attributes}. Given a decision system $\langle U, C, D \rangle$, let $R \subseteq C$ and $a \in C$ (but $a \notin R$) be a given attribute. The equivalence class that U is divided into under the attribute set R is $U/R = \{X_1, X_2, \dots, X_i, X_{i+1}, \dots, X_l\}$ and the equivalence class that U is divided into under the attribute a is $U/a = \{X'_1, X'_2, \dots, X'_k, X'_{k+1}, \dots, X'_s\}$. If $X_j \subseteq X'_t$ ($j = 1, 2, \dots, i, t = 1, 2, \dots, s$), the active region of a relative to R on U is represented as $ACT_a(R) = X_{i+1} \cup \dots \cup X_l$, and

the non-active region of a relative to R as $NACT_a(R) = X_1 \cup X_2 \cup \dots \cup X_i$. We only need to pay attention to $ACT_a(R)$ to determine whether a is a non-redundant attribute relative to the attribute set R .

Proof. let $U/R = \{X_1, X_2, \dots, X_i, X_{i+1}, \dots, X_l\}$, $NACT_a(R) = X_1 \cup X_2 \cup \dots \cup X_i$, $ACT_a(R) = X_{i+1} \cup \dots \cup X_l$.

Since $NACT_a(R)$ is the non-active region of a relative to R , we have

$$X_j \subseteq X'_t (j = 1, 2, \dots, i, t = 1, 2, \dots, s), \quad (11)$$

$$X_j \cap X'_t = X_j, \quad (12)$$

$$\begin{aligned} U/(R+a) &= \{X_1 \cap X'_1, X_1 \cap X'_2, \dots, X_2 \cap X'_1, \\ &\quad X_2 \cap X'_2, \dots, X_i \cap X'_1, \dots, X_l \cap X'_s\} \\ &= \{X_1, X_2, \dots, X_i, X_{i+1} \cap X'_1, X_{i+1} \cap X'_2, \dots, X_l \cap X'_s\} \\ &= \{NACT_a(R), X_{i+1} \cap X'_1, X_{i+1} \cap X'_2, \dots, X_l \cap X'_s\}. \end{aligned} \quad (13)$$

Focusing only on $ACT_a(R)$, i.e., that equivalence classes are divided on $ACT_a(R)$ and not on $NACT_a(R)$, we have,

$$\begin{aligned} U/(R+a) &= NACT_a(R) + ACT_a(R)/(R+a) \\ &= NACT_a(R) + \{X_{i+1} \cap X'_1, X_{i+1} \cap X'_2, \dots, X_l \cap X'_s\} \\ &= \{NACT_a(R), X_{i+1} \cap X'_1, X_{i+1} \cap X'_2, \dots, X_l \cap X'_s\}, \end{aligned} \quad (14)$$

which is the same as (26).

\Rightarrow Focusing only on $ACT_a(R)$ can determine the redundancy of a relative to the attribute set R .

Theorem 4 shows that, regardless of whether an attribute is redundant or not, its non-active region does not need to participate into further calculations, which results in a significant improvement for the performance. Here we use an example to demonstrate the theorem. Supposing that $U = \{x_1, x_2, \dots, x_8\}$ is a universe, $C = \{a_1, a_2, \dots, a_6\}$ is a set of conditional attributes containing six attributes and $D = \{d\}$ is a set of decision attributes. there is a decision system $\langle U, C, D \rangle$, as shown in Table I.

TABLE I
DECISION SYSTEM $\langle U, C, D \rangle$

	a_1	a_2	a_3	a_4	a_5	a_6	d
x_1	1	0	1	1	0	1	1
x_2	0	0	1	1	0	1	0
x_3	1	1	0	1	0	1	0
x_4	1	1	0	1	0	1	1
x_5	0	0	0	0	1	0	0
x_6	1	0	1	1	0	0	0
x_7	1	0	0	0	1	1	1
x_8	1	1	1	0	0	0	1

From Table I, we can calculate the following results.

$$\begin{aligned} U/D &= \{\{x_1, x_4, x_7, x_8\}, \{x_2, x_3, x_5, x_6\}\}, \\ U/C &= \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}\}, \\ POS_C(D) &= \{x_1, x_2, x_5, x_6, x_7, x_8\}. \\ U/(C-\{a_1\}) &= \{\{x_1, x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}\}, \\ POS_{C-\{a_1\}}(D) &= \{x_5, x_6, x_7, x_8\} \neq POS_C(D). \end{aligned}$$

$$\begin{aligned} U/(C-\{a_2\}) &= \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}\}, \\ POS_{C-\{a_2\}}(D) &= \{x_1, x_2, x_5, x_6, x_7, x_8\} = POS_C(D). \\ U/(C-\{a_3\}) &= \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}\}, \\ POS_{C-\{a_3\}}(D) &= \{x_1, x_2, x_5, x_6, x_7, x_8\} = POS_C(D). \\ U/(C-\{a_4\}) &= \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}\}, \\ POS_{C-\{a_4\}}(D) &= \{x_1, x_2, x_5, x_6, x_7, x_8\} = POS_C(D). \\ U/(C-\{a_5\}) &= \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}\}, \\ POS_{C-\{a_5\}}(D) &= \{x_1, x_2, x_5, x_6, x_7, x_8\} = POS_C(D). \\ U/(C-\{a_6\}) &= \{\{x_1, x_6\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}, \{x_7\}, \{x_8\}\}, \\ POS_{C-\{a_6\}}(D) &= \{x_2, x_5, x_7, x_8\} \neq POS_C(D). \end{aligned}$$

According to the above results, the core attribute set can be obtained as $\{a_1, a_6\}$.

$$U/\{a_1, a_6\} = \{\{x_1, x_3, x_4, x_7\}, \{x_2\}, \{x_5\}, \{x_6, x_8\}\},$$

$$POS_{\{a_1, a_6\}}(D) = \{x_2, x_5\}.$$

$$U/a_2 = \{\{x_1, x_2, x_5, x_6, x_7\}, \{x_3, x_4, x_8\}\},$$

$$\{x_2\} \subseteq \{x_1, x_2, x_5, x_6, x_7\}, \{x_5\} \subseteq \{x_1, x_2, x_5, x_6, x_7\}.$$

According to Definition 7, as $NACT_{a_2}(\{a_1, a_6\}) = \{x_2, x_5\}$, Theorem 1 can be used for acceleration and Theorem 4 is not needed. Thus, we use a_3 as an example to show the effectiveness of Theorem 4. Also based on Definition 7, $NACT_{a_3}(\{a_1, a_6\}) = \{x_2, x_5, x_6, x_8\}$, and the active region of attribute a_3 is $ACT_{a_3}(\{a_1, a_6\}) = \{x_1, x_3, x_4, x_7\}$. $\{x_2, x_5\}$ does not need to be considered in further calculations according to Theorem 1 because both of them belong to the positive region. $\{x_6, x_8\}$ do not need to be considered in later calculations according to Theorem 4 because they belong to $NACT_{a_3}(\{a_1, a_6\})$. Thus, the number of calculations is decreased after applying Theorem 4.

In addition, we compute $POS_{\{a_1, a_6, a_3\}}(D)$ according to whether $\{x_6, x_8\}$ are paid attention. If $\{x_6, x_8\}$ are paid attention, $POS_{\{a_1, a_6, a_3\}}(D) = \{x_1\}$; otherwise, $POS_{\{a_1, a_6, a_3\}}(D) = \{x_1\}$. Thus, $POS_{\{a_1, a_6, a_3\}}(D)$ remains to be the same whether $\{x_6, x_8\}$ are paid attention or not. So, this case demonstrates the effectiveness of Theorem 4.

IV. A ROBUST MEASUREMENT OF EVALUATING THE IMPORTANCE OF AN ATTRIBUTE FOR IMPROVING ACCURACY OF ROUGH SET

A. Motivation

In addition to the unfavorable performance, another reason preventing the rough set from being widely used is its less competitive classification accuracy. In this section, we show that the rough set is not robust to one kind of noise attribute, which widely exists in real data sets to different degree. This phenomenon can be described as follows.

Table II consists of the information about a group of students. As shown in this table, C_1, C_2, C_3 and C_4 are conditional attributes corresponding to the grades of the students. D is the decision attribute telling whether a student's overall performance is qualified. One can observe that C_1 containing a sequence of values is a noise attribute, which may come from the negligence of the statistician. However, in a rough set, this attribute can recognize each sample as a equivalence and a positive region sample, resulting in the overfitting issue. Therefore, in the reduction result, there only remains this attribute. Obviously, this reduction result is incorrect, and the equivalence rules of this case does not make sense — C_1 is

TABLE II
DECISION SYSTEM $\langle U, C, D \rangle$

C_1	C_2	C_3	C_4	D
1	3	4	5	1
2	3	4	5	1
3	4	3	2	1
4	3	3	2	0
5	5	5	5	0
6	4	3	2	1
7	4	3	2	1
8	3	3	2	0
9	5	5	5	1

such an extreme case. However, regardless of the presence of noise attributes, this phenomenon will occur to various degrees and lead to overfitting in the rough set. The specific reason is shown in Fig.1.

Fig.1(a) consists of five equivalences, seven positive region points and two boundary region points. After the attribute C_1 is added, although the number of positive region points increases from seven to nine, the number of equivalence classes increases from five to nine. It can be observed that the increased equivalence classes from the existing positive region equivalence classes are meaningless for the decision. In other words, more positive region points can decrease the boundary region, narrow the decision boundary and make the boundary clearer. This explains why the existing rough set methods utilize the number of positive region points as the measurement to evaluate the importance of an attribute. However, as shown in Fig. 1, more equivalence classes mean more rules and more complex boundaries, which is harmful for decision. It may lead to the overfitting issue, which is never considered in the existing rough set methods. Considering this problem, the relative importance is proposed to evaluate the importance of an attribute in the rough set. We consider both the number of positive region points and the number of equivalence classes.

C_2	C_3	C_4	D	C_1	C_2	C_3	C_4	D
3	4	5	1	1	3	4	5	1
3	4	5	1	2	3	4	5	1
4	3	2	1	3	4	3	2	1
3	3	2	0	4	3	3	2	0
5	5	5	0	5	5	5	5	0
4	3	2	1	6	4	3	2	1
4	3	2	1	7	4	3	2	1
3	3	2	0	8	3	3	2	0
5	5	5	1	9	5	5	5	1

(a)

(b)

Fig. 1. The equivalence results on the data set in Table (a) the equivalence results when noise attribute C_1 is not added. (b)the equivalence results when noise attribute C_1 is added. Blocks in the same color presents one equivalence belongs to the positive region. Grey represents the boundary region.

B. Relative importance of an attribute

Definition 8. Let $\langle U, C, D \rangle$ be a decision system. $\forall B \subseteq C$ and $a \in C - B$, the *relative importance* of a relative to B is

defined as,

$$RSIG(a, B, D) = \frac{\Delta |POS_B(D)|}{\Delta |U/B|} \quad (15)$$

$$= \frac{|POS_{B+a}(D)| - |POS_B(D)|}{|U/(B+a)| - |U/B|},$$

where $|\cdot|$ denotes the number of the elements in a set. Δ represents the increased amount. As shown in Definition 8, both the number of positive region points and equivalence classes can be considered. To be more specific, more positive region points and fewer equivalence classes provide us with a better result. For the first attribute selection, the initial number of positive region points and equivalence classes are both equal to 0. Because the acceleration strategies in Theorems 2 may affect the denominator of the relative importance model, we do not apply them when adopting the relative importance model.

V. AN EFFICIENT AND ACCURATE ROUGH SET FRAMEWORK

Based on the aforementioned theorems regarding the stability of redundancy attributes and the relative importance model, we propose the novel RSLRS framework to accelerate rough set methods. Here we provide a definition useful to structure our framework. As shown in Fig. 2, there are five main steps of our framework.

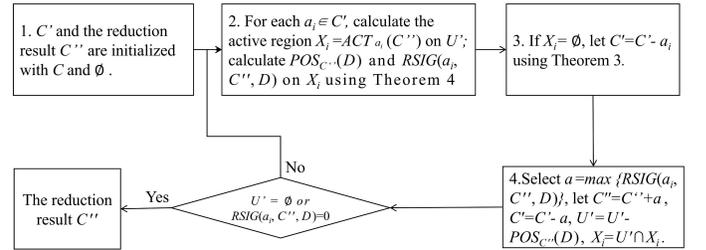


Fig. 2. A flow chart of the RSLRS framework

The first step is to initialize $C' = C, C'' = \emptyset, U' = U - POS_{C''}(D)$, and then comes the second step. For $\forall a_i \in C'$, we calculate the active region $ACT_{a_i}U'$ and $RSIG(a_i, C'', D)$ on $ACT_{a_i}U'$. According to Theorem 4, in the next step, we only need to decide whether there is a change in $ACT_{a_i}U'$, so as to reduce the repeated determination of samples.

Regarding the third step, if $ACT_{a_i}U' = \emptyset$, we remove a_i from C' using Theorem 3, obtaining that $C' = C' - a_i$. According to Theorem 3, the attribute a_i does not help to distinguish U' , thus it is a redundant attribute relative to the reduction result. By removing a_i from C' , we reduce the number of attributes in subsequent iterations. At this step, we achieve a further acceleration on the reduction algorithm for the classical rough set attributes.

At the fourth step, we select an attribute a for C' , which has the largest value of $RSIG(a_i, C'', D)$. A larger value of $RSIG(a_i, C'', D)$ will lead to a stronger representation ability. We then add the selected attribute a to the non-redundant attribute set C'' and remove a from C' , such that $C'' = C'' \cup a$ and $C' = C' - a$. Thereafter, we calculate the lower approximation of the updated C'' on U' (i.e., the

positive region of C'' on U') and update the data set U' using Theorem 1 such that $U' = U' - POS_{C''}(U')$. On the basis of updating the data set U' , we update $ACT_{c_i}U'$ and have $ACT_{c_i}U' = U' \cap ACT_{c_i}U'$. Therefore, the active region of an attribute is decreased while the later calculation is further accelerated.

After performing the above four main steps, we proceed to the next step based on whether U' or $RSIG(a_i, C'', D)$ is an empty set or not. If U' is not an empty set, we repeat step 2 as well as the following steps until either of the two conditions can be reached. Following the flowchart outlined in Fig. 2, we present the algorithmic details in Algorithm 1, where we propose the rough set using the model of relative importance and stability of local redundancy of attributes. We call it as relative stability of local redundancy rough set (RSLRS).

Algorithm 1 Attribute reduction algorithm

Input: A decision system $\langle U, C, D \rangle$;

Output: The reduction result C'' ;

- 1: Initialize $C' = C, C'' = \emptyset$;
 - 2: **for** $a_i \in C'$ **do**
 - 3: Calculate $ACT_{a_i}(C')$, and both $POS_{a_i+C''}(D)$ and $RSIG(a_i, C'', D)$ on $ACT_{a_i}(C')$; //Apply Theorem 4.
 - 4: **if** $ACT_{a_i}(C') = \emptyset$ **then**
 - 5: $C' = C' - a_i$; //Apply Theorem 3.
 - 6: **end if**
 - 7: **end for**
 - 8: Select $a = \max\{RSIG(a_i, C'', D)\}$; //Select the attribute with the biggest relative importance from among the remaining attributes.
 - 9: **if** $RSIG(a, C'', D) > 0$ **then**
 - 10: $C'' = C'' \cup \{a\}$;
 - 11: $C' = C' - a$;
 - 12: $U = U - POS_{C''}(D)$; //Remove the positive region under a from U using Theorem 1.
 - 13: **else**
 - 14: Return C'' .
 - 15: **end if**
 - 16: **if** $U \neq \emptyset$ **then**
 - 17: Go to Step 2;
 - 18: **else**
 - 19: Return C'' .
 - 20: **end if**
-

VI. KNOWLEDGE REPRESENT AND DATA CLASSIFICATION USING ROUGH SET

Rough sets can extract and generate knowledge rules. However, at present, there is no effective way to organize or to express the knowledge rules obtained by rough sets. To remedy this deficit, we present a knowledge representation method for rough set in this section. Take the data set of a disease as an example. The reduction result using Algorithm 1 is shown in Table 1, in which only three attributes (i.e., A, B and C) are selected. The importance of them are B, A and C respectively. The three attributes in the data set

represent the physical sign level data, and the +1 and -1 in the decision attribute indicate being diseased or non-diseased. Inspired by the concept lattice [33], we propose the knowledge representation called rough concept tree(RCT) for rough sets, as shown in Fig. 3. In the RCT, each node represents a "knowledge point" or "concept node", which consists of two parts — a sequence with both attributes whose values are called as "intent", and its corresponding equivalence called "extent" as the second part. These two parts are corresponding to that in the first row, first column and that in the second row, first column, respectively. In each "concept node", the "extent" is corresponding to the objects in an equivalence and the "intent" is corresponding to some attributes. Their values are borrowed from concept lattice. Besides, in a concept node, the value in its second row and second column counts the number of objects in its extent, and the value in its first row and second column represents the label of the extent, i.e., the equivalence class. If this value is a number, it suggests that the equivalence class belongs to the positive region and this concept node can describe a certain knowledge. Otherwise, the value is a question mark, indicating that the equivalence class belongs to the boundary region and can not describe a certain knowledge. As for the second layer of RCT in Fig. 3 corresponding to the attribute B, the second node represents a knowledge point (or a "concept node") which consists of an extent (i.e., an equivalence class) with the 1th, 9th and 12th points when the value of the attribute B is equal to 2, and the number of objects in the equivalence class is equal to 3. The equivalence class belongs to the positive region and its label is equal to "+1". Similarly, the second node in the third layer represents a knowledge point with an equivalence class containing the 5th, 6th, 10th and 14th points when the value of the attributes B and A are equal to 4 and 2, respectively. The question mark in this node represents that the equivalence class belongs to the boundary region. The RCT is ordered by the importance of attributes. The higher an attribute is, the greater the importance will be. Since the RCT is ordered, the extent corresponding to a given intent is fixed.

TABLE III
TAKE A DISEASE DATA AS AN EXAMPLE

Objects	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	1	1	1	1	2	2	1	1	2	1	1	1	1	2
B	2	4	5	5	4	4	4	4	2	4	3	2	4	4
C	1	1	1	1	3	3	1	1	2	1	1	1	1	3
Label	+1	-1	+1	+1	-1	-1	-1	-1	+1	+1	-1	+1	-1	+1

In a CRT, as shown in Fig. 3, all non-leaf nodes belong to boundary region, i.e., the nodes containing a question mark, indicating that the labels of the objects in a non-leaf node are not identical. Some leaf nodes belongs to the positive region, and others belongs to the boundary region, such as the green node in Fig. 3. In classification problems, as mentioned above, the leaf nodes belonging to the positive region can clearly describe the certain knowledge. The more objects in the extent and the fewer attributes in the intent in a concept node, the stronger the corresponding knowledge representation ability will be. Take Fig. 3 as an example. The two red nodes have the highest knowledge representation ability. On the one

hand, regarding the two equivalence classes, the extents in the two nodes can describe the largest collection of objects, so their generalizabilities are among the best; On the other hand, the fewer attributes can lead to a higher generalizability when other conditions are close to each other. The scale of rough concept tree is much smaller than the concept lattice, which overcomes the inefficiency of the concept lattice method. In summary, we provide a rough concept tree to organize and describe the knowledge rules obtained from forward rough sets. In addition, it is worth noting that, to be convenient for display in some large-scale RCTs, the names of attributes in the first row and first column of each concept node can be neglected because the RCT is ordered as shown in Fig. 4, and the attributes can thus be easily derived.

As shown in Fig. 3, RCT can be considered as a special decision tree. The rough set can be used for decision in classification problem according to the decision process, although RCT is not a rigorous decision tree because its nodes contain intent and extent instead of only objects. However, in the previous work [34], since only the equivalence classes in the positive region classes describe certain knowledges, they are used for decision in classification. However, this will not utilize the uncertain information in equivalence classes belonging to boundary region though the information is also useful. Therefore, its classification accuracy is low. In this paper, learning from the general decision tree algorithm, we also use knowledge information of the boundary region for decision by introducing the voting mechanism. Using the green node belonging to boundary region as an example, its decision label equals to the majority label in its extent, i.e., -1.

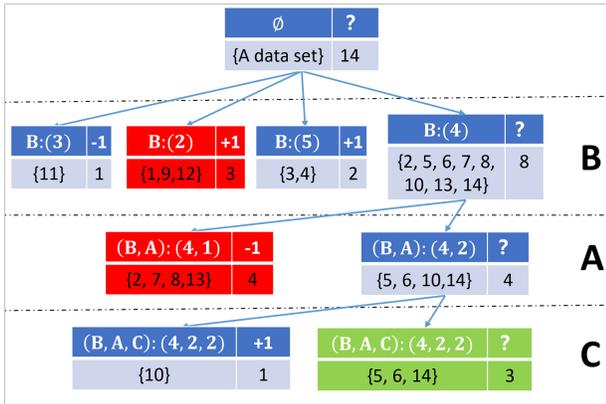


Fig. 3. Knowledge representation of rough concept tree and rough set classifier

Another challenge of the classification problem using rough sets is that how a classification task is completed using the RCT when any values of a new point in a test data set do not appear in the training set. For example, we need to predict the label of a person whose data values on attributes A, B and C are equal to (4.2,2.3,3) respectively using the RCT in Fig. 3. However, the value 4.2 does not appear in the attribute A, and the value 2.3 does not appear in the attribute B. To address this problem, for a value that does not appear in the existing data, we can use the closest value to approximate and replace it. Given a data set U containing n points and

d dimensionality and a point $(C_1^i, C_2^i, \dots, C_d^i) \in U$, for an unknown point (C_1, C_2, \dots, C_d) , its values can be transformed using the follow mathematical model:

$$f(C_1, C_2, \dots, C_d) = (C_1^*, C_2^*, \dots, C_d^*), s.t. \begin{cases} C_1^* = \min \{dis(C_1^i, C_1)\} \\ C_2^* = \min \{dis(C_2^i, C_2)\} \\ \dots \\ C_d^* = \min \{dis(C_d^i, C_d)\} \end{cases}, i=1, 2, \dots, n, \quad (16)$$

where $(C_1^i, C_2^i, \dots, C_d^i) \in U, i = 1, 2, \dots, n$. Using the value 4.2 in attribute A as an example, its closest value in attribute A is equal to 4, and in attribute B is equal to 2. Therefore, (4.2, 2.3, 3) is transformed to be as (4,2,3), and its label is equal to -1 according to RCT in Fig. 3 using the voting mechanism. The rough set classifier (RSC) is a rare classifier with good interpretability. As far as we know, logistic regression (LR) is the only classifier with this characteristic. Considering that LR is widely used in financial sector and other field, we include LR for comparison with RSC in the next section. In addition, RSC can provide an interpretability different from LR using the RCT shown in Fig. 3.

VII. EXPERIMENTS

In this section, we compare our algorithm with other seven widely-used or state-of-the-art attribute selection algorithms on ten benchmark datasets randomly selected from the UCI machine learning library. Relevant metadata of these data sets are summarized in Table V. We choose seven baseline algorithms, including two rough set attribute selection algorithms and six other attribute selection algorithms. The former two algorithms include the classical rough set algorithm and its acceleration version using Theorem 1. Only the classical one is used for comparison in accuracy because both of them have the same attribute selection results. The latter six algorithms include CFS [23], Fisher [20], ILFS [35], [40], Laplacian [41], LASSO [37], and Mutinffs. All these six algorithms use the implementation provided in the Feature Selection Code Library (FSLib) [40] (<https://www.mathworks.cn/matlabcentral/fileexchange/56937-feature-selection-library>). Regarding all eight algorithms for comparison, 10-fold cross-validation is used to evaluate the reduction results, and the average of the five classification accuracies is used as the final result.

A. Effectiveness of RSLRS in attribute selections

The experimental results of different attribute selection methods in accuracy are shown in Table IV. It can be observed that the RSLRS can provide a higher accuracy than other methods on most data sets, especially when comparing to CRS, because the model of relative importance evaluation can alleviate the overfitting issue in a rough set. Therefore, the RSLRS presents a higher generalizability than other rough set methods.

TABLE IV
CLASSIFICATION ACCURACY OF FRPOS

	cfs	fisher	ilfs	laplacian	lasso	mutinffs	RSLRS	CRS
anneal	0.9249±0.0161	0.9249±0.0161	0.9311±0.0161	0.9249±0.0161	0.9086±0.0200	0.9249±0.0161	0.9600±0.0151	0.8609±0.0097
Healthy_Older_People2	0.9717±0.0113	0.9724±0.0112	0.9717±0.0113	0.9311±0.0277	0.9311±0.0277	0.9724±0.0112	0.9739±0.0116	0.9561±0.0145
hepatitis	0.8134±0.0546	0.8071±0.0639	0.8001±0.0564	0.8134±0.0546	0.8134±0.0546	0.8071±0.0639	0.8326±0.0746	0.7867±0.0800
htru2	0.9706±0.0042	0.9734±0.0045	0.9713±0.0043	0.9713±0.0044	0.9736±0.0037	0.9734±0.0045	0.9761±0.0037	0.9759±0.0033
lymphography	0.6964±0.1158	0.7954±0.0938	0.8095±0.0885	0.7254±0.1245	0.7593±0.0902	0.7954±0.0938	0.8353±0.0935	0.7961±0.1324
zoo	0.8281±0.0877	0.8700±0.0806	0.8955±0.0957	0.8775±0.0867	0.8193±0.0780	0.8700±0.0806	0.9214±0.0880	0.8833±0.0542
letter	0.6908±0.0083	0.6250±0.0111	0.6827±0.0117	0.5462±0.0128	0.6540±0.0119	0.6250±0.0111	0.7148±0.0105	0.6539±0.0107
GINA	0.8512±0.0242	0.8718±0.0222	0.8668±0.0199	0.8281±0.0202	0.8322±0.0189	0.8674±0.0166	0.8135±0.0243	0.7380±0.0274
ionosphere.csv	0.8415±0.0655	0.8386±0.0605	0.8446±0.0657	0.8445±0.0722	0.8384±0.0658	0.8386±0.0605	0.7813±0.0623	0.7813±0.0623
sensorReadings.csv	0.5565±0.0545	0.6241±0.0321	0.6454±0.0385	0.6146±0.0240	0.6252±0.0295	0.6241±0.0321	0.6083±0.0412	0.6253±0.0228
vowel.csv	0.4305±0.1500	0.4359±0.1575	0.4177±0.1596	0.4345±0.1651	0.3468±0.1489	0.4359±0.1575	0.3982±0.1505	0.3977±0.1670

TABLE V
DATASET INFORMATION

Data sets	Features	Samples
anneal	21	798
Healthy_Older_People2.csv	9	22646
hepatitis.csv	20	155
htru2	9	17898
lymphography.csv	19	148
zoo.csv	17	101
letter	17	20000
GINA	971	3153
ionosphere.csv	33	351
sensorReadings	25	5456

TABLE VI
ACCURACY COMPARISON BETWEEN ROUGH SET CLASSIFIER AND LOGIC REGRESSION

	LR	RSC
anneal	0.8609	0.8271
Healthy_Older_People2	0.9561	0.9285
hepatitis	0.7867	0.7221
htru2	0.9759	0.9620
lymphography	0.7961	0.7167
zoo	0.8833	0.9409
letter	0.6539	0.8362
GINA	0.7380	0.6492
ionosphere	0.7813	0.8576
sensorReadings	0.6253	0.8198
vowel	0.3977	0.5622
ave	0.7687	0.8020

B. Effectiveness of rough set classifier and knowledge representation

Table VI presents the accuracies of each method. It can be observed that the RSC has a higher accuracy than others, including LR, on approximately half of the data sets. The knowledge nodes in the RCT consisting of both "intent" and "extent", as mentioned above, are very useful in many fields. Using the data set zoo as an example, there are sixteen conditional attributes including hair, feathers, eggs, milk, air, aquatic, predator, teeth, backbone, breath, venom, fins, legs, tail, whether it is domestic or not and catsize. The decision attribution of the zoo data set is the animal type such as mammals, fish, birds, invertebrates, insects, amphibians and reptiles, whose corresponding values range from 0 to 6. The RCT on the zoo is shown in Fig. 4. In the RCT, all the knowledge nodes without a question mark can describe the certain knowledge to determine the animal type. Among them, the three knowledge nodes in red contain the most objects and have the largest knowledge description ability. For a new data point of an animal, if its third conditional attribute, i.e., milk, is equal to 1, we can use this knowledge point to assign it to the first animal type, i.e., mammals. In summary, the animals that produce milk are all mammals. Similarly, the other two red knowledge nodes indicate that an animal which does not produce milk with 2 legs is a bird; an animal with no legs but with fins which does not produce milk is a fish. These statements are easy to understand and to validate from a biological point of view.

C. Efficiency of RSLRS

In this section, we compare the efficiency among multiple popular rough set algorithms. For a fair comparison in efficiency between different acceleration strategies in the rough set, the relative importance model is used in all the three rough set algorithms so that all of them have the same reduction results. The efficiency results of the experiment are shown in Fig. 4. Logarithm of running time is used to make all the results easier to visualize. It can be observed that the efficiency of the RSLRS is significantly improved in comparison with other acceleration methods for rough sets because our RSLRS framework can considerably decrease the number of equivalence classes calculation in computation for positive region using the stability of local redundancy.

VIII. CONCLUSION

This paper presents an effective framework to accelerate the rough set and improve its accuracy by proposing the theorem regarding the stability of the local redundancy of attributes and the model of relative importance. Besides, we develop the rough set to be a classifier and knowledge presentation tool. To the best of our knowledge, this work has enabled the rough set to be the only data mining method inherently effective in feature selection, classification, and knowledge representation. The experimental results validate that the proposed method can

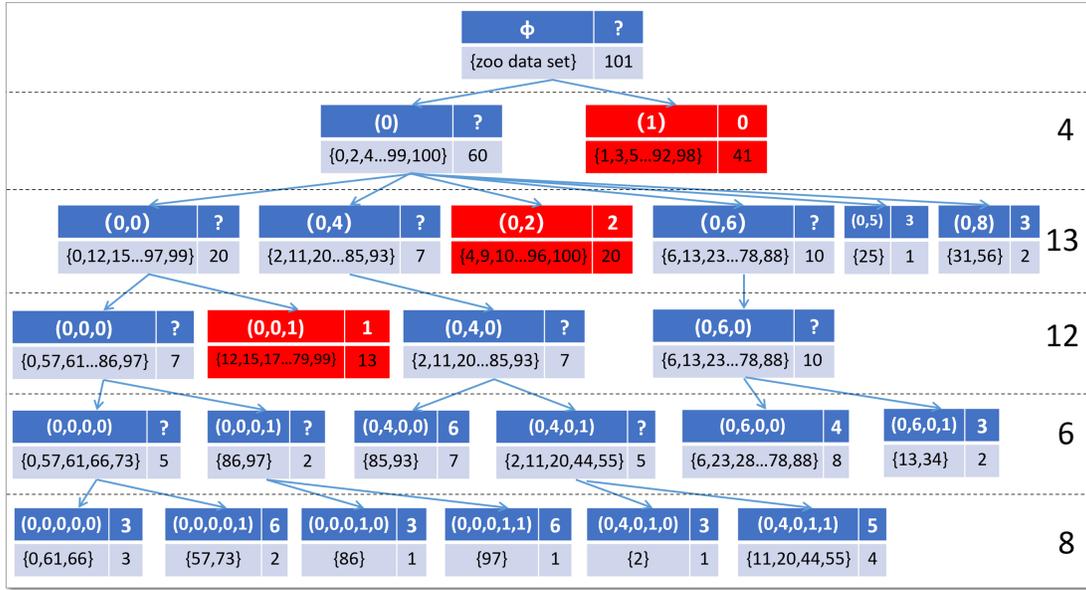


Fig. 4. Knowledge representation of the data set zoo using rough concept tree.

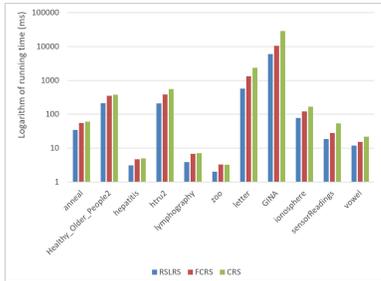


Fig. 5. Efficiency comparison

achieve a higher accuracy compared with the state-of-the-art attribute selection algorithms in most cases. We also show that our method is always more efficient than other acceleration rough set methods. This work significantly improves the ability of rough sets in feature selections when being applied in real-world scenarios. Future work will focus on further improving its efficiency for large-scale input data by parallelizing the current framework.

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